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Localized tachyon condensation and G-parity conservation

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Abstract: We study the condensation of localized tachyon in non-supersymmetric orbifold $\mathbb{C}^2/\mathbb{Z}_n$. We first show that the G-parities of chiral primaries are preserved under the condensation of localized tachyon(CLT) given by the chiral primaries. Using this, we finalize the proof of the conjecture that the lowest-tachyon-mass-squared increases under CLT at the level of type-II string with full consideration of GSO projection. We also show the equivalence between the G-parity given by $G = [jk_1/n] + [jk_2/n]$ coming from partition function and that given by $G = \{jk_1/n\}k_2 - \{jk_2/n\}k_1$ coming from the monomial construction for the chiral primaires in the dual mirror picture.

Keywords: Renormalization Group, Conformal Field Models in String Theory, Tachyon Condensation.

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Contents

1. Introduction

After discovering interesting phenomena on the tachyon condensation in open string theory [1], it has been tantalizing question to ask the same in closed string cases. The simplest closed string tachyon is the case where the closed string tachyon is localized at the singular point of the background geometry. In this direction, Adams, Polchinski and Silverstein [2] considered localized tachyon for non-compact orbifold $\mathbb{C}^r/\mathbb{Z}_n$ and argued that starting from a non-supersymmetric orbifolds, there will be a cascade of tachyon condensation until space-time SUSY is restored. In a subsequent paper, Vafa [3] reformulated the problem using the mirror picture of gauged linear sigma model, which turns out to be an orbifolded Landau-Ginzburg theory, and confirmed the result of APS.

Since the tachyon condensation process can be considered as a renormalization goup (RG) flow [4], it would be interesting to ask whether there is a quantity like a c-function. In non-compact orbifolds, the c-theorem [6] does not work [2, 5]. Therefore the authors of [5] tried to establish a closed string analogue of the g-theorem of boundary conformal field theory. It turns out that, if valid, it would give an explicit counter-example to the result of APS. On the other hand, in a related paper [8], Dabholkar and Vafa suggested that the minimal R-charge in the Ramond sector is the height of tachyon potential at the unstable critical point.

In [9], it is argued that the g_{cl} of [5] does not respect the stability of supersymmetric theory and suggested a modified quantity to replace it. In a subsequent paper [10], one of the present authors suggested that the minimal tachyon mass squared should increase under the localized tachyon condensation. It turns out that this quantity is nothing but the GSO projected version of (negative of) minimal R-charge of Dabholkar-Vafa mentioned above. Later, the statement has been studied in a series of the papers [11, 12, 13], and it is proved that the R-charge decreases at the level of conformal field theories before GSO projection. For type-II theory, the proof was incomplete mainly due to the incomplete understanding of behavior of G-parity of GSO projection under the tachyon condensation. In related papers [12] the picture of Vafa was extended by working out the generators of daughter theories (the result of decay of the mother theory). The chiral rings and GSO-projection of orbifold theory was examined in more detail in [13].

The goal of this paper is to complete the proof of statement with full consideration of GSO projection, namely to prove the following statement for type-II theory of $\mathbb{C}^2/\mathbb{Z}_n$ orbifold:

- Let $m := \max\left|\alpha' M^2\right|$. Then, $m(UV) \geq m(IR)$, under condensation of localized tachyon.

Notice that $M^2_{\rm min}$ is negative and increases under the tachyon condensation while m is positive and decreases as stated above. Figure 1 is the schematic diagram to clarify the content of this statement. We call this as a m-theorem to prevent possible confusion with c-theorem or g-theorem.

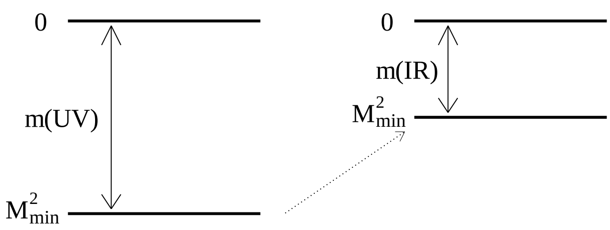

Figure 1: $M^2_{\rm min}$ increases and m decreases under the localized tachyon condensation.

The rest of the paper goes as follows. In section 2, we consider the consistency of various GSO-projections introduced by arbitrarily different authors using different logics. In section 3, we show that, $n(k_1, k_2)$, the $\mathbb{Z}_n$ orbifold with generator (k_1, k_2) equivalent to $n(1, k)$ for some k which we will fix in detail. We call the latter as "canonical representation". In section 4, we prove that the G parity is conserved under the condensation of the localized tachyon given by the chiral primaries. This is the most important step in proving the m-theorem. In section 5, we finish the proof of the m-theorem. In section 6, we give a discussion on the implication of the theorem and conclude.

2. Equivalence of various GSO-projections

In this section we first want to understand whether orbifold GSO chiral projections recently introduced in [3, 5, 10] are mutually consistent. Let k_1, k_2 be the generator of the orbifold action of $\mathbb{Z}_n$, that is,

$$x^{(1)}(z) \to e^{2\pi i k_1/n} x^{(1)}, \qquad x^{(2)}(z) \to e^{2\pi i k_2/n} x^{(2)}, \tag{2.1}$$

where $x^{(1)}, x^{(2)}$ are complex co-ordinate of $\mathbb{C}^2$. We represent $\mathbb{C}^2/\mathbb{Z}_n$ with generator (k_1, k_2) by $n(k_1, k_2)$. In HKMM [5], $k_1 = 1$ cases were discussed. Here we discuss their result in the extended form, that is, with general (k_1, k_2). The chiral primary operators were constructed from bosonized world sheet fermions $\psi_i = e^{iH_i}$ as

$$X_j = X^{(1)}_{n\{\frac{jk_1}{n}\}} X^{(2)}_{n\{\frac{jk_2}{n}\}}, \tag{2.2}$$

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where

$$X_j = \sigma_{\frac{j}{n}} e^{i\frac{j}{n}(H-\bar{H})}\,, \tag{2.3}$$

with $\sigma_{j/n}$ being a twist operator. The Z_2 action defining the GSO projection is given by

$$H_1 \to H_1 + k_2\pi\,, \qquad H_2 \to H_2 - k_1\pi\,. \tag{2.4}$$

In untwisted sector it acts as $(-1)^{F_L}$ and restricts both k_1 and k_2 to be an odd. In twisted sector X_j has phase

$$X_j \to e^{i\pi(k_2\{\frac{jk_1}{n}\}-k_1\{jk_2/n\})}X_j := (-1)^s X_j\,, \tag{2.5}$$

where $\{x\} = x - [x]$ with $[x]$ being the greatest integer that does not exceed x. Note that s is an integer in general and especially when $k_1 = 1$ and $k_2 = k$

$$s = \left[\frac{jk}{n}\right]. \tag{2.6}$$

In [3], Vafa reformulated the orbifold problem as Landau-Ginzburg theory by imbedding the orbifold geometry in the gauged linear sigma model [14] and subsequently taking the mirror dual. The superpotential coming from the vortex contribution can be written as

$$W = u_1^n + u_2^n + e^{t/n}u_1^{p_1}u_2^{p_2} \tag{2.7}$$

$(-1)^{F_L}$ should be defined by requiring $W \to -W$. This can be achieved by defining the Z_n action on u_i by[1]

$$u_1 \to e^{i\pi k_2/n}u_1\,, \qquad u_2 \to e^{-i\pi k_1/n}u_2\,. \tag{2.8}$$

As a result,

$$u_1^{p_1}u_2^{p_2} \to (-1)^{p_1k_2/n-p_2k_1/n}u_1^{p_1}u_2^{p_2} := (-1)^{s'}u_1^{p_1}u_2^{p_2}\,. \tag{2.9}$$

So by identifying $p_i = n\{jk_i/n\}$, $i = 1, 2$, we get

$$s = p \times \frac{k}{n} = s'\,, \tag{2.10}$$

and consequently, two GSO actions are completely consistent. Notice that s is always an integer. We also see that $u_1^{p_1}u_2^{p_2}$ in the mirror LG theory corresponds to X_j of the operator construction.

We now want to see whether the GSO projection coming from the partition function [10, 13] is also consistent with above two. The result of ref. [10] shows that $u_1^{p_1}u_2^{p_2}$ is projected out if

$$G = \left[\frac{jk_1}{n}\right] + \left[\frac{jk_2}{n}\right] \tag{2.11}$$

is even (odd) for *cc* ring (*ac* ring). For our purpose, it is enough to show that

$$G \equiv s \bmod 2\,. \tag{2.12}$$

[1] We have to make modifications of discussion of [3] on GSO projection.

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j	cc	G	s	ca	j	cc	G	s	ca	j	cc	G	s	ca
1	$(2,4)$	0	0	$(2,1)$	1	$(1,2)$	0	0	$(1,3)$	1	$(1,2)$	-1	-1	$(1,3)$
2	$(4,3)$	1	2	$(4,2)$	2	$(2,4)$	0	0	$(2,1)$	2	$(2,4)$	-2	-2	$(2,1)$
3	$(1,2)$	3	0	$(1,3)$	3	$(3,1)$	1	1	$(3,4)$	3	$(3,1)$	-2	-2	$(3,4)$
4	$(3,1)$	4	2	$(3,4)$	4	$(4,3)$	1	1	$(4,2)$	4	$(4,3)$	-3	-3	$(4,2)$

Table 1: Comparison of G and s in $\mathbf{5(2,4)}$ (left) $\mathbf{5(1,2)}$ (middle) and $\mathbf{5(1,-3)}$ (right). $\mathbf{5(2,4)}$ (left) provides an example where $s \neq G$ mod 2. However, $s \equiv G$ mod 2 for all elements in $\mathbf{5(1,2)}$ and $\mathbf{5(1,-3)}$.

We first notice that for special case $k_1 = 1$, $k_2 = k$,

$$s = \left[\frac{jk}{n}\right] = G\,. \tag{2.13}$$

In the next section we will prove that all $n(k_1, k_2)$ have equivalent representation $n(1, k)$ for some k. So the above result is enough if we consider only a given theory. However, we will need to consider the case where $k_1 \neq 1$ when we consider the 'decay' of $n(1, k)$ by explicitly specifying the daughter theories. So let's consider the general cases. For type II, we need to have $k_1 + k_2 =$ even [13]. If both k_1, k_2 are odd integers, the equivalence can be readily seen by considering $s + G$ with the help of the following identity.

$$s = k_2\left\{\frac{jk_1}{n}\right\} - k_1\left\{\frac{jk_1}{n}\right\} = -k_2\left[\frac{jk_1}{n}\right] + k_1\left[\frac{jk_2}{n}\right], \tag{2.14}$$

so that $s + G$ is even, which is enough for our goal. If both k_1, k_2 are even, then s is even and G is not necessarily equivalent to s. In table 1, we give an explicit example for this case. However, if we further restrict ourselves to the case where n, k_1, k_2 are mutually co-prime, we can restrict ourselves to the case where k_1, k_2 are odd. Later in section 4, we will only need to consider the case where $k_1 = 1$ or both k_1, k_2 are odd.

3. Equivalence of $n(k_1, k_2)$ and $n(1, k)$

Now, we want to show that $n(k_1, k_2)$ is equivalent with $n(1, k)$ for some k. We can choose a convention where $k_1 > 0$, since $n(k_1, k_2) = n(-k_1, -k_2)$ even after GSO projection.[2] First, notice that (k_1, k_2) and $(1, k)$ should generate the same spectra if $k_2/k_1 = k$ and n, k_1 and n, k_2 are relatively co-prime, since the spectrum is nothing but the modulo-n-rearrangement of $j(k_1, k_2)$ for $j = 1, \ldots, n-1$. In fact, any of the element of the spectrum, that is, any of $j(k_1, k_2)$ modulo n can be the generator of the same spectrum set. Therefore, without GSO projection, the equivalence of the two is quite obvious. For type 0 case, the GSO projection does not eliminate any variety of chiral primaries in the following sense: if an operator with a certain charge is projected out in cc ring, there is a surviving operator in aa ring with the same charge. This can be seen from the fact that j-th element of

[2] One can see this from $[-jk_i/n] = -[jk_i] - 1$ regardless of the sign of k_i.

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cc-ring and $(n-j)$-th element of aa ring have the same charge but different G-parity if $k_1+k_2 = \text{odd}$ [13]:

$$n\left(1-\left\{\frac{(n-j)k_i}{n}\right\}\right) = n\left\{\frac{jk_i}{n}\right\}, \qquad G(n-j) \equiv G(j)+k_1+k_2 \bmod 2\,. \tag{3.1}$$

Similar relations hold between ca and ac rings. Therefore if two type 0 theories have the same spectrum before GSO projection, so do they after GSO.

Hence from now on, we concentrate on the type-II case, where $k_1+k_2 = \text{even}$. We first have to specify k more precisely. Let k_1^{-1} be the multiplicative inverse of k_1 in $\mathbb{Z}_n$ so that there is a unique integer a depending on k_1 such that

$$k_1^{-1}k_1 = na+1\,, \qquad \text{for any given } k_1\,. \tag{3.2}$$

Then, k is equal to $k_1^{-1}k_2$ modulo n. So there exists an integer l such that

$$k = k_1^{-1}k_2 + ln\,, \qquad \text{and} \qquad -n<k<n\,. \tag{3.3}$$

In fact, there are two such l's, since the length of range is $2n$. They are consecutive. In order for $n(1,k)$ to be a type-II string theory, we require that

$$k = k_1^{-1}k_2 + ln = \text{odd}\,. \tag{3.4}$$

For odd n, this fixes l uniquely, since $k \neq k \pm n \bmod 2$. However, for even n, the ambiguity will be removed only after we take account the G-parity of k^{-1} and l more carefully. Before we proceed, we give some examples to give some feeling on how things work.

- $8(3,-5)$: it is the same with $8(1,1)$ before GSO projection, since $-5\cdot 3^{-1} \equiv 1$ in $\mathbb{Z}_8$. However, these are NOT equivalent after GSO as one can see from the table 2. Then, it may look like a counter example. However, for even n, both $n(k_1,k_2)$ and $n(k_1,k_2\pm n)$ represent the same type of theory regarding to whether they are type 0 or type II [13]. Here $\pm$ is chosen such that $-n<k_2\pm n<n$ is satisfied. Therefore we should also consider $8(1,-7)$ instead of $8(1,1)$. Remarkably, $8(3,-5)$ and $8(1,-7)$ have the same GSO projected spectrum as one can see from table 2.

- $7(3,5)$: it is equivalent to $7(1,-3)$ and also to $7(1,4)$ before GSO projection. But $7(3,5)$ and $7(1,-3)$ is a type II while $7(1,4)$ is type 0. Therefore in this case there is a unique representation in the same type. One can explicitly check that $7(3,5)$ has identical spectrum with $7(1,-3)$ after GSO from table 3.

Now let us come back to the general argument. Consider G-parity for $n(k_1,k_2)$ and $n(1,k)$ with k given in eq. (3.3). We call them as A and B orbifold theory respectively. The G-parity for j-th element of A is $G_A(j) = [jk_1/n]+[jk_2/n]$ and the G-parity for j-th one of B is $G_B(j) = [jk/n]$. We remind that the cc- and ca-rings of A and B are the same (as sets) before GSO projection. Let the j-th element of A theory appears as the j'-th element for B so that

$$j' = n\{jk_1/n\}. \tag{3.5}$$

j	cc	G	s	ca	j	cc	G	s	ca	j	cc	G	s	ca
1	$(5,5)$	-1	5	$(5,3)$	1	$(1,1)$	0	0	$(1,7)$	1	$(1,1)$	-1	-1	$(1,7)$
2	$(2,2)$	0	2	$(2,6)$	2	$(2,2)$	0	0	$(2,6)$	2	$(2,2)$	-2	-2	$(2,6)$
3	$(7,7)$	-1	7	$(7,1)$	3	$(3,3)$	0	0	$(3,5)$	3	$(3,3)$	-3	-3	$(3,5)$
4	$(4,4)$	0	4	$(4,4)$	4	$(4,4)$	0	0	$(4,4)$	4	$(4,4)$	-4	-4	$(4,4)$
5	$(1,1)$	1	1	$(1,7)$	5	$(5,5)$	0	0	$(5,3)$	5	$(5,5)$	-5	-5	$(5,3)$
6	$(6,6)$	0	6	$(6,2)$	6	$(6,6)$	0	0	$(6,2)$	6	$(6,6)$	-6	-6	$(6,2)$
7	$(3,3)$	1	3	$(3,5)$	7	$(7,7)$	0	0	$(7,1)$	7	$(7,7)$	-7	-7	$(7,1)$

Table 2: cc-,ca-ring of $\mathbf{8(-3,5)}$ (left), $\mathbf{8(1,1)}$ (middle), $\mathbf{8(1,-7)}$ (right). G, s for each element are given for comparison. Both $8(1,-7)$ and $8(1,1)$ are equivalent to $8(-3,5)$ before GSO. But only $8(1,-7)$ is so after GSO. This is a general phenomena: for even n type II, k is not determined uniquely from (k_1,k_2) before GSO. This ambiguity or freedom will be essential to find correct k with GSO projection considered.

j_1	cc-elements	G	ca-elements	j_1	cc	G	ca
1	$(3,5)$	0	$(3,2)$	1	$(1,4)$	-1	$(1,3)$
2	$(6,3)$	1	$(6,4)$	2	$(2,1)$	-1	$(2,6)$
3	$(2,1)$	3	$(2,6)$	3	$(3,5)$	-2	$(3,2)$
4	$(5,6)$	3	$(5,1)$	4	$(4,2)$	-2	$(4,5)$
5	$(1,4)$	5	$(1,3)$	5	$(5,6)$	-3	$(5,1)$
6	$(4,2)$	6	$(4,5)$	6	$(6,3)$	-3	$(6,3)$

Table 3: cc-, ca-rings for $\mathbf{7(3,5)}$ (left) and $\mathbf{7(1,-3)}$ (right). Notice that 7(1,4) is a type 0 theory and it is not tabulated here. For odd n type II, k is chosen uniquely from (k_1,k_2).

For our purpose, it is enough to show that

$$G_A(j) \equiv G_B(j') \bmod 2. \tag{3.6}$$

Since k is an odd integer,

$$G_B = \left[\left(\frac{jk_1}{n} - \left[\frac{jk_1}{n}\right]\right)k\right] \equiv \left[\frac{jk_1}{n}\right] + \frac{jk_1(k_1^{-1}k_2 + ln)}{n} \bmod 2\,. \tag{3.7}$$

Using eq. (3.2), one can easily show that

$$G_B \equiv G_A + jk_1 l + jk_2 a \bmod 2\,. \tag{3.8}$$

If both k_1, k_2 are even and n is odd, then our job is done. If both are odd, then $G_B \equiv G_A + j(l+a)$ modulo 2. For even n, one of the consecutive l's (see below eq. (3.3)) can be chosen such that $l+a$ is even and this condition removes the ambiguity in the choice of l (hence in k) as mentioned before.

For odd n, l is fixed as follows: if $k_1^{-1}k_2$ is already odd, then l should be even not to change the type 0/type II. If the former is even, then l should be odd. For $k_1^{-1}k_2$ odd case, using eq. (3.2),

$$(na+1)k_2 = k_1 \cdot \text{odd} \equiv k_1 \bmod 2\,. \tag{3.9}$$

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If k_1, k_2 are both odd, $na+1$ should be odd. Since n is odd, a must be even. Therefore if $k_1^{-1}k_2$ is odd and k_1, k_2 have the same number of factor 2, $l+a$ is even as desired. Similarly, for $k_1^{-1}k_2$ even case,

$$(na+1)k_2 = k_1 \cdot \text{even} \equiv k_1 \bmod 2\,. \tag{3.10}$$

If k_1, k_2 are both odd, then $na+1$ must be even, hence a must be even. Therefore in this case also $l+a$ is even as desired. Hence we proved the following

Lemma. *For any orbifold $n(k_1, k_2)$, we can represent it by $n(1, k)$ with GSO projection properly considered.*

4. Conservation of G-parity under LTC

The final most important step in proving the m-theorem is to show that G parity is conserved under the localized tachyon condensation. Namely, if a particular charge is projected out in the mother theory, its daughter image under T_p is also projected out in a daughter theory.

In the previous work [12], we showed that the decay of $n(k_1, k_2)$ under the condensation of localized tachyon with weight $p = (p_1, p_2)$ is

$$n(k_1, k_2) \to p_1(k_1, s) \oplus p_2(-s, k_2) \tag{4.1}$$

where $p_1 = \{jk_1/n\}$, $p_2 = \{jk_2/n\}$ and $s = p \times k/n = k_2\{jk_1/n\} - k_1\{jk_2/n\}$. Its G-parity is given by

$$G_p = \left[\frac{jk_1}{n}\right] + \left[\frac{jk_2}{n}\right] \bmod 2 \tag{4.2}$$

Here $[x]$ means the integer part of x while $\{x\}$ means the fractional part of x. In order for (p_1, p_2) to survive, G_p should be an odd integer. On the other hand, by the result of last section, we only need to consider the decay of a canonical representation:

$$n(1, k) \to p_1(1, s_p) \oplus p_2(-s_p, k)\,, \tag{4.3}$$

with $p_i = n\{jk_i/n\}$, $s_p = p \times k/n = (p_2 - kp_1)/n$. In order to fix the ambiguity in the daughter-theory-generators, we use the fact that for type-II theory, the bulk tachyon is projected out and it can not and should not be regenerated by the localized tachyon condensation process. That is, type II can decay only to type II. Then s and k should be both odd. This conditions are already satisfied: the orginal theory is type II hence k is odd and p as a surviving element of cc-ring must have odd $G_p(= s_p)$. If any of p_1, p_2 is odd, that can not be added or subtracted to the generator, since it convert the type II to type 0. If any of them are even, it can be added to the generator and we have ambiguity in the determination of the daughter theory. However, as we will see shortly, this can not be so and we will see that the eq. (4.3) describe the tachyon condensation properly even after the GSO projection. For a moment, we assume that eq. (4.3) is true.

We need the map which gives the G-parity value $[lk/n]$ when the data $(q_1, q_2) = (l, n\{lk/n\})$ of a charge are given. This is given by the observation:

$$G_q = \left[\frac{lk}{n}\right] = \frac{(kq_1 - q_2)}{n} = q \times \frac{k}{n}\,. \tag{4.4}$$

Under the condensation of (p_1, p_2), q is mapped to q' by the tachyon map T_p [11]. If q' belongs to up-theory of daughter theory,

$$q' = T_p^+ q = \begin{pmatrix} 1 & 0 \\ -p_2/n & p_1/n \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} q_1 \\ p \times q/n \end{pmatrix} = \begin{pmatrix} q_1' \\ q_2' \end{pmatrix} . \tag{4.5}$$

Then,

$$G_{q'} = \frac{q_2' - q_1' s_p}{p_1} = \frac{1}{np_1}(p \times q - q_1(p \times k)) = \frac{1}{n}(q \times k) = G_q \,, \tag{4.6}$$

which proves the conservation of G-parity for the up-theory spectrum. Similarly we can prove the same statement for the down-theory using the equivalence of G-parity with s proved before. Notice that both s and k are odd, so that the result applies here. Since

$$q'' = T_p^- q = \left(-p \times \frac{q}{n}, q_2\right) := (q_1'', q_2'') \,, \tag{4.7}$$

we have

$$G_{q''} = \frac{q_2'' s + q_1'' k}{p_2} = \frac{1}{np_2}(-(p \times q)k + q_2(p \times k)) = \frac{1}{n}(q \times k) = G_q \,, \tag{4.8}$$

finishing the proof of G-parity conservation.[3]

An important remark is in order. The decay rule eq. (4.1) or eq. (4.3) is given at the conformal field theory level. After GSO, since $n(k_1, k_2)$ and $n(k_1, k_2 \pm n)$ are different theories, it seems that the decay rule has the same ambiguity: that of adding $\pm p_1$, $\pm p_2$ to s and k_2 respectively in the right hand side of eq. (4.1): why the following process is not allowed?

$$n(k_1, k_2) \to p_1(k_1, s \pm p_1) \oplus p_2(-s, k_2 \pm p_2) \,. \tag{4.9}$$

If $p_1(k_1, s_p)$ is replaced by $p_1(k_1, s_p \pm p_1)$, then a computation shows that

$$G_{q'} = G_q \mp q_1 \,. \tag{4.10}$$

This means that an operator that is projected out in the mother theory can be resurrected in daughter theory under the localized tachyon condensation(LTC). This is not physical, because what changes under the LTC is not the operator $u_1^{q_1} u_2^{q_2}$ but the lagrangian:

$$W = u_1^n + u_2^n + e^{t/n} u_1^{p_1} u_2^{p_2} \,. \tag{4.11}$$

Namely, each operator remains the same and only the coefficient $e^{t/n}$ of condensing tachyon operator changes so that the measuring method of the weight of operators $u_1^{q_1} u_2^{q_2}$ changes. Once an operator is projected out at the moment of $t \to -\infty$, it is not possible to resurrect it by a smooth deformation of taking $t \to \infty$. The same argument holds for the $p_2(-s_p, k \pm p_2)$. Therefore the original rule eq. (4.1) or eq. (4.3) remains its form and the modified rule eq. (4.9) is not allowed.

[3] We can also use the inverse representation $n(k', 1) \to p_1(k', s_p') \oplus p_2(-s_p', 1)$, to prove the G-parity conservation for the down theory spectrum. Here $k' := k^{-1}$ in the finite field $\mathbb{Z}_n$, and $s_p' = p \times k'/n = (p_1 - k' p_2)/n$. Using $G = [lk'/n] = (q_2 k' - q_1)/n$, we have $G_{q'} = \frac{q_2'(-s_p') - q_1}{p_2} = \frac{1}{np_2}(-q_2(p \times k') + p \times q) = \frac{1}{n}(q_2 k' - q_1) = G_q$.

5. Proof of m-theorem

According to the result of [11], the spectrum of daughter theories can be regarded as images of certain linear mapping T for cc ring and F for ca or ac rings given by

$$\begin{aligned} T_p^+(q) &= \left(q_1, p \times \frac{q}{n}\right), & T_p^-(q) &= \left(-p \times \frac{q}{n}, q_2\right), \\ F_p^+(\bar{q}) &= \left(q_1, p_1 - p \times \frac{q}{n}\right), & F_p^-(\tilde{q}) &= \left(p_2 + p \times \frac{q}{n}, q_2\right). \end{aligned} \tag{5.1}$$

Here, superscript $+$ is for up-theory, $-$ is for down-theory and $\bar{q} = (q_1, n - q_2) \in ca$ ring, $\tilde{q} = (n - q_1, q_2) \in ca$ and $p' = (p_1, -p_2)$. T is a mapping for tachyon condensation which exists due to the worldsheet $\mathcal{N} = 2$ SUSY, while F is not a mapping describing tachyon condensation and its existence is due to the conformal symmetry of the final theory. For more detail, see [11]. The most important property of T and F is their monotonicity, which enables the proof of m-theorem possible.

The conservation of G parity means that the same logic can be applied to the spectrum of type II even after GSO projection. The worst thing that can happen is the case where the operator with minimum R-charge of the Mother theory is deleted in the mother theory but resurrected in the daughter theory. The G-parity conservation guarantees that such phenomena can never happen. See figure 2.

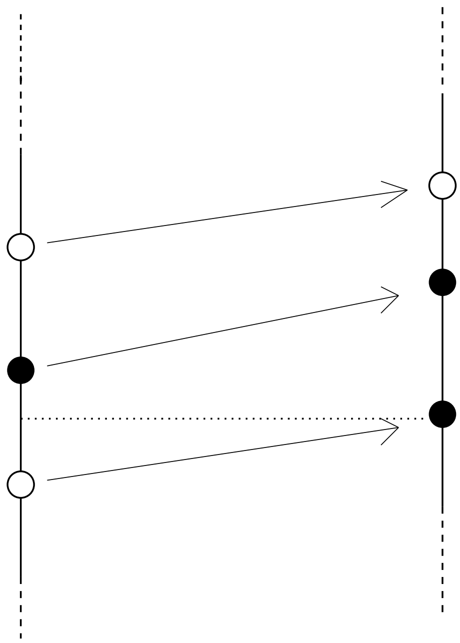

Figure 2: A candidate phenomena that can destroy the m-theorem in the absence of the G-parity conservation. Filled dots represent operators surviving under the GSO projection and empty dots are those projected out.

For completeness, we describe some of the detail here. Let p be the condensing tachyon and $q'_{\rm min}$ be the element with minimal R-charge of the daughter theory, i.e. the smallest one among all charges in up and down theories. If $q'_{\rm min}$ belongs to cc ring of up-theory, then it is an image of an element q in the mother theory under T_p^+. q must belong to the cc-ring of the mother theory due to the G-parity conservation. If $q_{\rm min}$ is the minimal element of the mother theory, $R[q_{\rm min}]$, the R-charge of $q_{\rm min}$, is smaller than $R[q]$ by definition. Then our desired statement,

$$R[q_{\rm min}] \leq R[q'_{\rm min}] \tag{5.2}$$

comes from the monotonicity of T^+, namely,

$$R[q] \leq R[T_p^+(q)] = R[q'_{\rm min}]\,. \tag{5.3}$$

If $q'_{\rm min}$ belongs to cc ring of down-theory, then T_p^- replaces T_p^+ for above argument. If $q'_{\rm min} \in ca$-ring of daughter theories, then the entire arguments can be repeated by replacing the map $T^\pm$ by $F^\pm$ to finish the proof.

6. Discussion

Our proof was actually motivated from the numerical work we performed, which showed that the theorem holds for all $n(k_1, k_2)$ with $n \leq 100$ and $-n < k_1 < k_2 < n$, which provided significant evidence to believe that the m-theorem is true.

The m-theorem we just proved implies that the one loop cosmological constant of the non-supersymmetric orbifold is a monotonically decreasing quantity under the localized tachyon condensation. This is because, the former is defined as the integral of one loop partition function over the fundamental domain with some cutoff to control the divergence. The main divergence comes from the low temperature limit of the partition function which is dominated by the contribution of the lowest tachyon mass. Therefore the present work proves the statement of the original conjecture of [10] in its full strength. The cosmological constant vanishes when the theory reaches a supersymmetric point by the tachyon condensation.

We describe some future work. Our work is still comparing two end points of RG where conformal invariance makes the explicit computations available. Therefore it is still needed to extend our work to the string off-shell level, which would require much more non-trivial efforts. Some preliminary work in this direction for the simplest case is discussed in Dabholkar and Vafa [8]. Extension in this direction is in progress [15].

Acknowledgments

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